Project 5: Cooperative and Competitive Species

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**Abstract**

Many cooperative and competitive systems can be modeled by relatively simple first order differential equations. In this project, we aim to explore different types of interspecies relationships. In particular, we will look at cooperative models, in which both species mutually benefit from the other (such as a sea anemone and a clown fish), and competitive relationships, in which both species are harmed by interaction with the other (such as pythons and alligators in the Florida Everglades). To analyze such systems, we used mostly numerical and graphical techniques. Phase plots and null-clines of the system were used to study and predict the behavior of the populations over time. In our study we explore the effects of harvesting on one of the populations in a cooperative system, as well as the long term behavior of competitive and cooperative systems.We found that as the harvesting rate increases the two equilibrium points move farther away from each other. In the long term, the populations of these systems will tend towards one of the sink or spiral sink equilibrium points.

**Main Body**

1. \*In the book\*

System A is cooperative. The first term in the dx/dt equation ( -5x) represents the growth of population x in absence of population y. Likewise, the first term in the dy/dt equation (-4y) represents the growth of population y in absence of population x. The second term in the dx/dt equation (2xy) gives the effect on population x due to population y, while the second term in the dy/dt equation (3xy) gives the effect on population y due to population x. The growth rate coefficients of population depend on the populations involved. System A is cooperative because if either population dies out the other will too. We know this because the first term of each equation is a negative exponential and the second term is positive.

System B and C are competitive. In dx/dt, the term -4xy shows the effect of population y on the rate of change of population x, while the dy/dt term, -2xy, shows the effect of the x population on the rate of change of the y population. The remaining terms in either equation actually combine to form a logistic growth model. We know these systems are competitive because -4xy and -2xy are negative. The negative sign shows that the populations are competing for the same resource. The populations can survive without each other. Population x will grow according to the logistic model without population y and vice versa.

1. \*In the book\*

In System A the equilibrium points are (4/3, 5/2) and (0,0). The equilibrium point (0,0) is a sink and the equilibrium point (4/3,5/2) is a saddle point.

In System B the equilibrium points are (0,0), (0, 3/2), (5/2, 0), and (4/3, 7/6). (0,0) is a source and the other equilibrium points are spiral sinks.

In System C the equilibrium points are (0,0), (0,6), (10, 0), and (14/3, 4.3). (0,6) and (10,0) are saddles, (0,0) is a source and (14/3, 4/3) is a sink.

\* See graphs for vector fields and solution curves

1. \*In the book\*

System A:

When y < 4/3 and x < 5/2, both derivatives have a positive trajectory and increase toward (4/3, 5/2) and away from (0,0).

When y < 4/3 and x > 5/2, population x increases while population y decreases.

When y > 4/3 and x < 5/2, population y increases while population x decreases.

When y > 4/3 and x < 5/2, both derivatives have a negative trajectory and decrease towards (0,0) and away from (4/3, 5/2).

System B:

When y > (3/2) and x > 0, y > 0 and x > 5/2, y > 7/6 and x > 4/3 then the x and y populations will decrease toward their respective equilibrium point.

When 0<y< 3/2 and x < 0 then the y population increases and the x population decreases towards (0, 3/2).

When y < 0 and 0 < x < 5/2 population x increases and population y decreases towards (5/2, 0).

When population x and y are zero they won’t grow.

System C:

When y > 6 and x < 0, populations x and y decrease.

When y < 6 and x < 0, population y increase and population x decreases.

When 0 < y < 6 and x > 0, populations x and y increase towards (14/3, 4/3).

When y > 4/3 and x > 14/3, populations x and y decrease towards (14/3, 4/3).

When y > 4/3 and 0 < x < 14/3, population x increases and y decreases towards ( 14/3, and 4/3).

When x and y are zero, the populations don’t grow.

In the long term, the populations of these systems will tend towards one of the sink or spiral sink equilibrium points.

1. \* From Blackboard\*

If there was harvesting rate added to the dx/dt equation of System A the new equation would be -5x+2xy-h where h is an arbitrary constant equivalent to the amount harvested. For the purpose of simplicity we graphed the new equation where h = 5. In the new graph there is still two equilibrium points. The equilibrium point on the x -axis is a sink. On the line y = 1 the vector fields move away in opposite directions, where x is positive and y > 1 the populations are increasing, but while y < 1 and x is negative the populations are decreasing. All the observations are similar to the system with no harvesting rate, however the population changes with harvesting are more dramatic (shown by the vector fields having a higher slope). As the harvest rate increases, the equilibrium points move further and further apart. We found that the bifurcation value for system A in the x equation is -6.9 however this would never happen because harvesting values are always positive.

**Conclusion:**

Exploring cooperative and competitive systems is important because it describes our ecosystem. We as humans harvest populations when we hunt for food or harvest vegetation. We are directly affecting populations that participate in other cooperative or competitive systems, therefore we must be aware of the long term effects we have on the populations around us so we can better decide at what rate to harvest or consume prey at. We noticed that as we increased the harvest rate of population x in the cooperative system, the equilibrium points moved further away from each other. This tells us that as the x population decreases, a higher y population is needed to establish a system in equilibrium. We know now that in a competitive logistic system the populations can survive without each other. Population x will grow according to the logistic model without population y and vice versa. (See appendix) The growth coefficients just affect when an equilibrium system can be reached.

**Appendix**

See attached pages